

Solar activity in the past and the chaotic behaviour of the dynamo

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Abstract The record of solar activity is reviewed here with emphasis on peculiarities. Since sunspot positions tell us a lot more about the solar dynamo than the various global sunspot numbers, we first focus on the records of telescopic observations of sunspots leading to positional information. Then we turn to the proxy record from cosmogenic isotope abundances, which shows recurrent grand minima over the last 9500 years. The apparent distinction between episodes of strong modulation, and intervening episodes with milder modulation and weaker overall activity, hints at the solar dynamo following a variety of solutions, with different symmetries, over the course of millennia.

1 Introduction

Telescopic observations of sunspots have revealed both the 11-year Schwabe cycle and the interruption of activity associated with the Maunder Minimum in the 17th century. New analyses of early records (including those of Schwabe) confirm that the pattern associated with the butterfly diagram has been present for the past 300 years. There is also evidence of differential rotation, with suggestions of anomalous behaviour during the Maunder Minimum.

The record of solar activity has been extended back for almost 10 000 years by measuring the abundances of the cosmogenic isotopes ^{10}Be and ^{14}C in ice

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cores and tree rings. This record reveals many grand minima, with a characteristic spacing of around 200 years (the de Vries cycle) but the appearance of these grand minima itself varies with a characteristic timescale of over 2000 years. We interpret the grand minima and maxima as resulting from deterministic modulation of the nonlinear solar dynamo, oscillating chaotically with the mean Hale period of around 22 years. The present peculiar evolution of the solar cycle may turn out to be an enlightening period in this respect as well, since it will be observed by various methods including helioseismology.

Although the Sun’s magnetic field is now predominantly dipolar, simple nonlinear models show that symmetry can flip to give quadrupolar or even mixed-mode behaviour. We suggest that such flipping explains the long-term, multimillennial variability of the activity record.

2 The sunspot record

The sunspot number goes back to Wolf (1859) who defined an index of solar activity based on the number of sunspot groups and the total number of individual spots on the observable hemisphere of the Sun. The time series starts in 1749 with the observations by Johann Staudacher (Nuremberg) and has been continued in terms of the International Sunspot Number until the present day. An alternative index was defined by Hoyt and Schatten (1998) who only counted the group numbers – an index that is more robust against variable capabilities of seeing small spots and allowed the time series to go back to the first days of telescopic observations of the Sun in 1610.

The time-series are very often used as a proxy for some sort of magnetic field strength in the interior of the Sun and are compared with dynamo models. These global indices, however, cannot give any insight into the topology of the magnetic fields that presumably generate the spots on the surface. One may imagine that the knowledge of the heliographic positions of the spots can be used to infer the equatorial symmetry, the latitudinal distribution, and the rotational symmetry of the underlying magnetic fields as well as the lifetime of magnetic structures on the solar surface.

Sunspot positions are now being collected in a database using results from a USAF network of observing stations, following on the photoheliographic programme conducted by the Royal Greenwich Observatory (RGO) which effectively also was a network of stations around the world. The RGO/USAF set started in 1874 and stores only the average position of sunspot groups together with their total area. In parallel, several other programmes were collecting sunspot positions of various amounts, or are still doing so.

Before that, a large set of sunspot data is available from Friedrich Wilhelm Gustav Spörer who observed from 1861 to 1894 from the towns of Anklam and Potsdam, Germany. He confirmed that the lull of reports on sunspots in the second half of the 17th century represented a real low in solar activity for decades. His work was recognized later by Maunder who was eventually credited with that discovery, whence the name “Maunder minimum”. Spörer’s

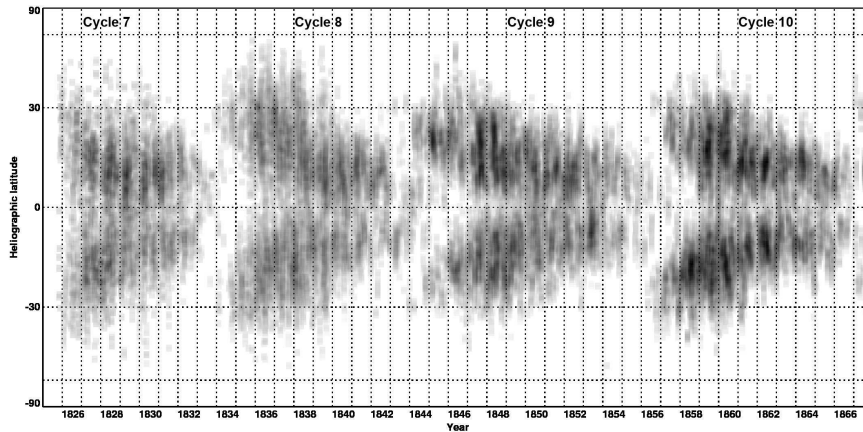


Fig. 1 Butterfly diagram obtained from the sunspot positions derived from the drawings of Samuel Heinrich Schwabe in 1825–1867. (After Arlt et al. 2013.)

observations and measurements were published in a series of papers (Spörer 1874, 1878, 1880, 1886, 1894), but his original sunspot drawings – if they existed – are lost.

Before Spörer, Richard Carrington had already made the discovery that the Sun is not rotating uniformly, but faster at the equator than at higher latitudes (Carrington 1859). However, his observational data only cover a rather short period, from November 1853 to March 1861 (Zolotova et al. 2010, Lepshokov et al. 2012).

A great extension of the butterfly diagram into the past comes from the observations by Samuel Heinrich Schwabe, who drew sunspots in a solar disk each day he saw at least a glimpse of the Sun from Nov 5, 1825, to Dec 31, 1867. He actually observed until Dec 15, 1868, but his last observing book was lost. Schwabe was also the first to publish a paper suggesting that the sunspot number varies periodic (Schwabe 1844). The positions and sizes of all sunspots seen by Schwabe were measured by Arlt et al. (2013). The resulting butterfly diagram is shown in Fig. 1.

The time around 1800 is poorly covered by observations, the longest record being that preserved by Honoré Flaugergues, who reported useful sunspot observations in 1788–1830. These, however, are yet to be analysed. They consist mostly of timings at a transit instrument. Flaugergues gave transit times of the solar limb and spots at a vertical and an oblique wire, which will allow us to determine the latitudes of spots.

A very interesting record of observations is stored in the library of the Leibniz Institute for Astrophysics Potsdam. About 1000 drawings of the Sun made by Johann Caspar Staudacher cover the period of 1749–1799. The drawings are not accompanied by much verbal information about the telescope or the observing method. There are no indications of the orientations of the solar disks. Fortunately, detailed drawings of partial solar eclipses showing the

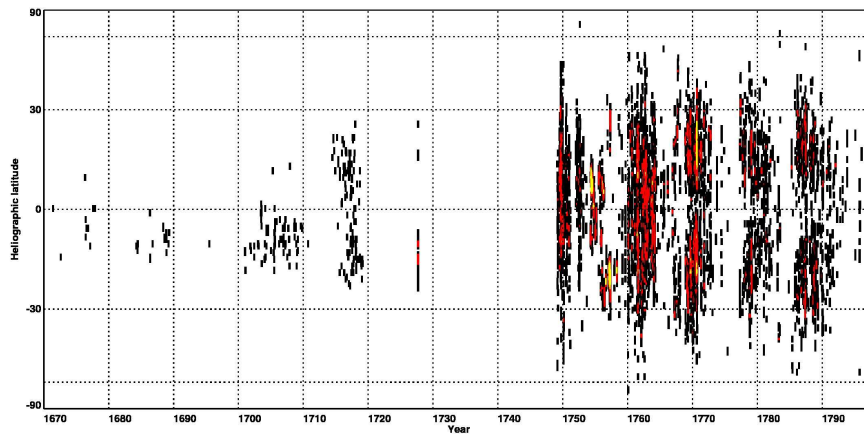


Fig. 2 Butterfly diagram of the period around and after the Maunder minimum with sunspot positions from various sources. The period until 1719 shows the positions derived by Ribes and Nesme-Ribes (1993) which were digitized by Vaquero et al. (<http://haso.unex.es/>); the year 1727 shows two additional observations found at Paris observatory by the authors. The period of 1749–1799 contains observations by Staudacher (Arlt 2009a), Zucconi (Cristo et al. 2011), and Hamilton (Arlt 2009b). Higher contrast is used than in Fig. 1 because of the fewer spots available.

path and direction of the Moon clearly show that the Sun was projected on a screen behind the telescope, i.e. all images are mirrored. The resulting butterfly diagram is shown in Fig. 2 (Arlt 2009a). It is remarkable that the first two cycles observed by Staudacher do not show a clear butterfly shape. Since the observer recorded the sunspots with the projection method, they are certainly not plotted ‘at random’ into the disks, though the uncertainty of the orientations holds true for the entire data set.

The observations were complemented by the very accurate drawings of Ludovico Zucconi in 1754–1760 (Zucconi 1760) which were analyzed by Cristo et al. (2011). They fill in the gaps of the observations by Staudacher around the minimum near 1755 and may help understand the unusual time–latitude distribution of Staudacher’s spots. At the other end of Staudacher’s observing period, additional positions of a small number of sunspots were derived from the records of Hamilton and Gimmingham at Armagh Observatory in 1795–1797 (Arlt 2009b).

Further back in time, we find the analysis by Ribes and Nesme-Ribes (1993) who measured the positions of sunspots seen by a variety of astronomers at the Observatoire de Paris, during and after the Maunder minimum, resulting in data for 1672–1719 (Fig. 2). There is a striking absence of sunspots until about 1714 with only the southern hemisphere being populated, by roughly 80 spots (plus 4 spots in the northern hemisphere and 6 spots right on the equator). This result again shows that the solar cycle does not necessarily show a butterfly shape for the time–latitude distribution of spots. The actual activity cycle may have persisted during the Maunder minimum as seen in

the cosmic ray record (see Sect. 4) at high time resolution (Beer et al. 1998; Berggren et al. 2009).

A fair number of observers recorded sunspots in the period before the Maunder minimum, starting with the first telescopic observations by Galileo Galilei and Thomas Harriot in 1610, followed by Christoph Scheiner and Johannes Fabricius, who first published the telescopic sunspot observations, in a printed pamphlet (Fabricius 1611). There is no compilation of sunspot positions for all the available sources yet, but visual inspection of the images indicates normal spot distributions before the Maunder minimum.

3 Results from the sunspot record

Sunspots show the latitudinal differential rotation of the Sun. This has first been derived quantitatively by Carrington (1859) and Peters (1859). The rotation of newly emerged sunspot groups is, by the way, different from the rotation of the bulk gas at the surface at the same latitude (Tuominen 1962, Pulkkinen and Tuominen 1998).

Historical sunspot observations may actually allow measurements of the differential rotation. A recent attempt by Arlt and Fröhlich (2012) employed Bayesian inference on the observations by Staudacher to obtain positions, orientation angles and differential rotation parameter at the same time and delivered a latitudinal shear compatible with that of today. There is a slight but insignificant hint that the differential rotation was stronger in the first third of Staudacher's observations than during the remaining period. This coincides with the period of non-butterfly shaped distribution of spot latitudes over time which is an automatic side product of the analysis.

The spot positions derived by Ribes and Nesme-Ribes (1993) also indicate a stronger differential rotation at the end of the Maunder minimum, along with an unusual spot distribution as well. There is actually no fundamental reason why the solar magnetic field should always adopt a chiefly dipolar structure (antisymmetric with respect to the equator). Quadrupolar modes or mixed symmetries are certainly possible and may lead to butterfly diagrams deviating from the one we know today.

Another issue of the solar cycle and the solar dynamo is the coupling between the hemispheres. Zolotova et al. (2010) studied the temporal variation of the phase lag between the cycles separated into hemispheres. The phase difference varies and changes sign approximately every 35–40 years, giving a full period for the phase lag change of 70–80 years.

4 The cosmic-ray record

Galactic cosmic rays are deflected by magnetic fields in the heliosphere and so the solar cycle modulates the flux of galactic cosmic rays into the Earth's atmosphere. The detection of the near-Earth cosmic rays by decay products

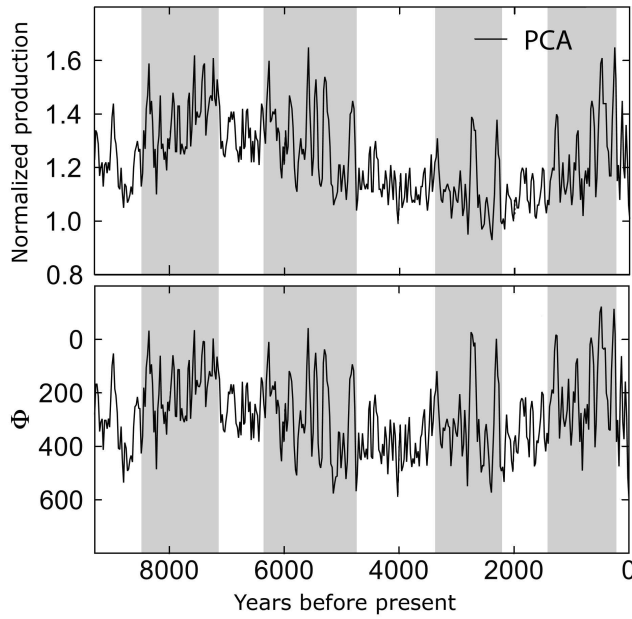


Fig. 3 The 9400-year record of solar magnetic activity derived from cosmogenic isotopes for the period from 9400 BP to the present, where ‘present’ means 1950. Upper panel: the principal components record of the production rate, based on the INTCAL09 record for ^{14}C and GRIP and EDML abundances for ^{10}Be . Lower panel: the modulation function Φ , in MeV, after correction in an attempt to eliminate the effects of variations in the dipole moment of the geomagnetic field. The shaded strips denote the intervals with vigorous modulation of solar magnetic activity, associated with the presence of prominent grand maxima and grand minima. (After McCracken et al. 2013.)

in the atmosphere (mostly neutrons) delivered a 60-year record which shows a very good anti-correlation with the sunspot record over the last five cycles – see Usoskin (2013) for a review. Cosmic rays also lead to the production of isotopes such as ^{14}C and ^{10}Be , which are absorbed into tree-rings or into polar icecaps, where their abundances can be measured with great precision. A 1000-year time-series of the isotope productions of ^{10}Be and ^{14}C (Muscheler et al. 2007) shows good agreement with the sunspot record, although some significant differences remain. More recently, the ^{14}C data have been combined with ^{10}Be measurements from Greenland and Antarctic ice cores to produce a much longer, composite time-series of cosmic radiation with a duration of 9400 years. A Principal Components Analysis has then been used to filter out most of the climatic effects and to reveal the presence of recurrent grand maxima and grand minima (Steinhilber et al. 2012; Abreu et al. 2013; McCracken et al. 2013).

The resulting time series can then be converted into a record of the modulation function Φ , which can be very roughly interpreted as the mean loss of momentum-to-charge ratio of a cosmic ray particle in the heliosphere (Usoskin

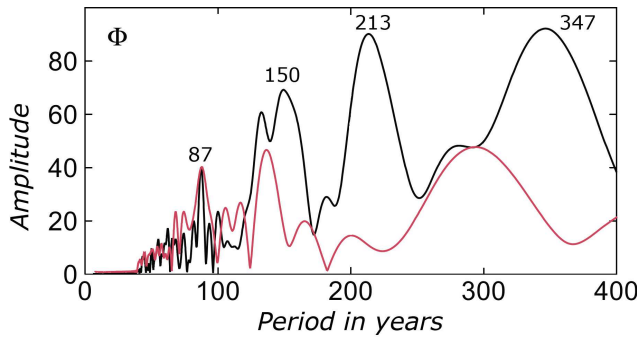


Fig. 4 Comparison of Fourier amplitude spectra for intervals with and without grand minima. Shown in black is the spectrum for Φ over the interval from 6300 to 4300 BP (containing four prominent grand minima). The spectrum in red is for the interval from 4700 to 3500, which contained no grand minima (as can be seen in Fig. 3). Apart from the Gleissberg peak at 87 yr and a possible coincidence around 135 yr, the two spectra have little in common. (After McCracken et al. 2013.)

2013). Fig. 3 displays the principal components rate of production of cosmogenic isotopes from 9400 BP to 1950 CE, together with the derived variation of the modulation function Φ , which has been corrected to take account of changes in the Earth's magnetic field (Knudsen et al. 2008).

The variations in the shaded regions of Fig. 3 are all similar to those in the most recent millennium, which includes the Maunder, Spörer, Wolf and Oort Grand Minima. Between these regions there are intervals, from 7100 to 6300 BP and from 4700 to 3500 BP, and again from 2200 to 1700 BP during which there are no grand minima and variations in isotope production are relatively low. This distinction becomes even more apparent if we compare the Fourier amplitude spectra for intervals with and without grand extrema, as displayed in Fig. 4. The only clear coincidence is for the Gleissberg cycle, with a period of 87 yr. The interval with strong modulation shows the familiar de Vries period of 208 yr and other peaks at 150 and 350 yr, while the 2300 yr Hallstatt period shows up in the full record. The intervals on either side, with only weak modulation, have a broad peak around 300 yr and a sharper peak at 140 yr but the spectra are generally flatter. All this confirms the immediate impression that the behaviours in the shaded and unshaded regions of Fig. 3 are qualitatively different. The Hallstatt period then represents the characteristic timescale for transitions from one regime to the other.

5 Chaotic modulation and symmetry-breaking

The Sun's cyclic activity is governed by macroscopic physics and is therefore deterministic. It is not practicable, however, to follow the emergence of every flux tube to form the sunspots that are shown in Fig. 1 and so we have to focus on averaged behaviour, which is deterministic but subject to stochas-

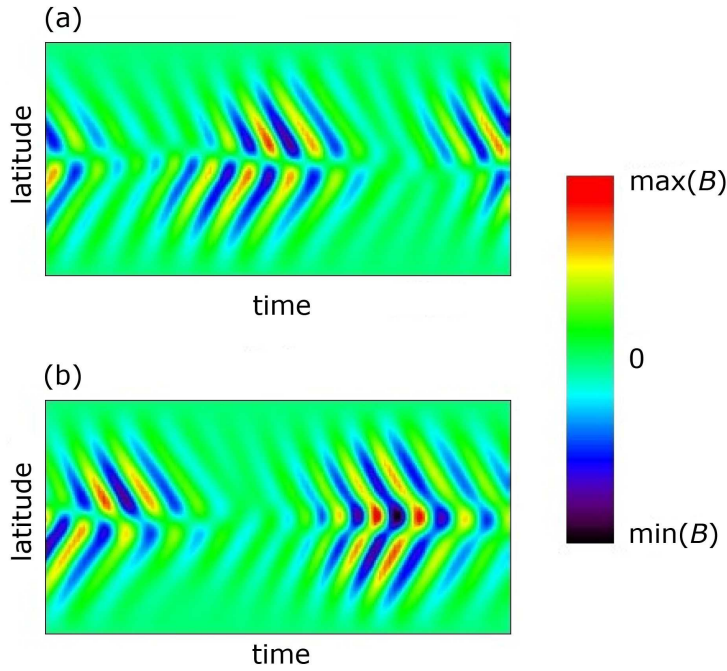


Fig. 5 Modulation and symmetry changes in a Cartesian mean-field dynamo model governed by partial differential equations, showing the toroidal field as a function of latitude and time. (a) Active fields with dipole symmetry, exhibiting grand minima associated with loss of symmetry and hemispheric patterns. (b) A transition from dipole symmetry to quadrupole symmetry during a grand minimum (for the same parameter values). (After Beer et al. 1998.)

tic disturbances. Then it is apparent that activity cycles must be regarded as manifestations of a chaotic oscillator, with sensitive dependence on initial conditions (e.g. Zeldovich et al. 1983; Spiegel 2009). The evolution of such a system is represented by a trajectory in phase space; provided the stochastic perturbations are not too large, the disturbed trajectories are always *shadowed* by nearby trajectories of the undisturbed chaotic system (Ott 1993). Turning to the observational records described above, we should therefore expect the chaotic system to generate modulation corresponding to grand minima and grand maxima, whose origin can be understood by reference to the mathematical structure of the problem (Tobias and Weiss 2007). Simple models do indeed reproduce similar behaviour, which is associated with the appearance of multiply periodic (“quasiperiodic” to mathematicians) solutions and global bifurcations that lead to chaos.

In the simplest illustrative model, cyclic dynamo action sets in at an oscillatory (Hopf) bifurcation that leads to periodic behaviour, with trajectories that are attracted to a limit cycle in the phase space; this is followed by a

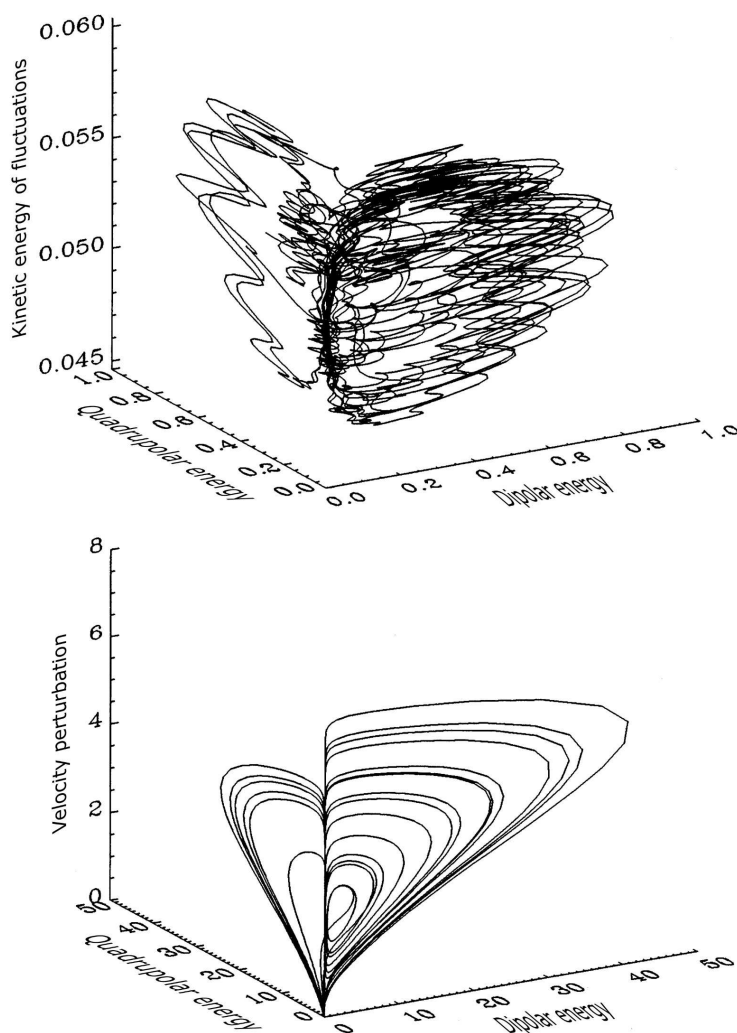


Fig. 6 Phase portraits illustrating flipping between dipole and quadrupole polarities for simple model systems. Upper panel: a trajectory for the PDEs, corresponding to Fig. 5 above, projected onto the 3-dimensional space spanned by the dipole energy, the quadrupole energy and the perturbed kinetic energy. Lower panel: the same but for the ODEs, with the perturbed velocity as the ordinate. In each case the symmetry flips occasionally at deep grand minima. (After Knobloch et al. 1998.)

second Hopf bifurcation that leads to doubly periodic solutions that lie on a 2-torus in the 3-dimensional phase space; after a series of period-doubling bifurcations (associated with a heteroclinic bifurcation) behaviour becomes chaotic (Tobias, Weiss and Kirk 1995), though the chaotic modulation still retains a memory of its original periodicity (Ott 1993). Since this bifurcation sequence was originally demonstrated for normal form equations (governing a

saddle-node/Hopf bifurcation) it is generic and therefore expected to be robust. Indeed, the same pattern appears in simple dynamo models governed by partial differential equations (Tobias 1996) and in mean-field dynamos (Küker et al. 1999; Pipin 1999; Bushby 2006). It follows that grand minima and grand maxima should be interpreted as *deterministic* effects, associated with chaotic modulation, and not as products of large-scale stochastic disturbances.

All dynamo models allow two families of solutions that bifurcate from the trivial, field-free state: these families have either dipole symmetry (with toroidal fields that are antisymmetric about the equator) or quadrupole symmetry (with symmetric toroidal fields). Mixed modes can only appear as a result of symmetry-breaking bifurcations in the nonlinear domain, which may lead to a complicated web of stable and unstable solution branches (Jennings and Weiss 1992). After the Maunder minimum, the solar magnetic field appears to have gained dipolar symmetry by the second half of the 18th century, through a period of mixed symmetry (with nearly all spots in one hemisphere only) and a period poorly covered by observations.

Similar properties are exhibited by an idealized mean-field dynamo model, governed by partial differential equations (Beer, Tobias and Weiss 1998), which also allows transitions between dipolar and quadrupolar symmetries during grand minima. This behaviour is shown in Fig. 5. These properties are also demonstrated by even simpler models, governed by low-order systems of ordinary differential equations (Knobloch et al. 1998). Phase portraits for both PDEs and ODEs are displayed in Fig. 5. The former show both cyclic variations (predominantly horizontal) and large-amplitude modulation, as well as occasional changes of symmetry. With the ODEs it is possible to filter out the cyclic variability, leaving only the modulation with flips of symmetry near the origin, at very deep grand minima. Note the reduced amplitude of modulation in the quadrupole regime as compared with dipole fields. By changing the parameters in the model systems it is possible to find mixed-mode cycles too; they are likewise modulated, and different symmetries may coexist without the possibility of flipping.

These results for highly simplified models reveal generic properties that would be shared by solutions of the much more complicated equations that describe a real stellar dynamo. What they show is not only that grand maxima and grand minima are associated with deterministic modulation of cyclic activity but also that symmetry changes may provide a natural explanation for the changes in behaviour that were reported by McCracken et al. (2013) and summarized in Sections 2–4 above. A more detailed discussion will appear elsewhere (Tobias and Weiss, in preparation).

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